## TEACHERS

## Ratio and Proportion

Ratio: Ratio is the comparison between two quantities in terms of their magnitudes. The ratio of two quantities $a$ and $b$ in some units, is the fraction $\frac{a}{b}$ and we write it as $a$ : $b$. In the ratio $a: b$, we call ' $a$ ' as the first term or antecedent and ' $b$ ', the second term or consequent.
Ex. The ratio 2:3 represents $\frac{2}{3}$ with antecedent $=2$, consequent $=3$

Example: A type writer takes 8 hours to type 960 pages. How many pages will it type in 12 hours?
Sol. $\frac{8}{960}=\frac{12}{x}$
$\mathrm{x}=1440$

## Rule

- The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
- The comparison should always be done of the same quantity (of length, of weight etc.)

Ex. $2: 3=4: 6=8: 9=10: 15$
Example: The two number are in ratio $2: 3$. If sum of the square two numbers is 20800 , find the numbers?
Sol. $4 \mathrm{x}^{2}+9 \mathrm{x}^{2}=20800$
$13 \mathrm{x}^{2}=20800$
$\mathrm{x}^{2}=1600$
$\mathrm{x}=40$
Ist no. $=80$, IInd no. $=120$

Proportion: When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional.
When $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ are in proportion, then a and d are called EXTREMES and $b$ and $c$ are called MEANS.
If $a: b=c: d$, we write, $a: b: c: d$ and say that $a, b, c, d$ are in proportion. Here a and b are called extremes, and b and c are called means terms.
Thus a:b :: c:d $\Rightarrow(\mathrm{a} \times \mathrm{d})=(\mathrm{b} \times \mathrm{c})$


Example: Which of the following numbers should be added to $13,43,23$, and 73 So that they are in a proportion?
Sol. by Adding 2
$=\frac{(13+2)}{(43+2)}=\frac{(23+2)}{(73+2)}$
$\frac{15}{45}=\frac{25}{75}$
$\frac{1}{3}=\frac{1}{3}$
So, the number $=2$

Example: find the value of ' a ' in the following proportion 36 : $108:: \mathrm{x}: 12$
Sol. $\frac{36}{108}=\frac{x}{12}$
$\mathrm{x}=4$

Fourth proportional: If 2:3 :: 5:7, then 7 is called the fourth proportional to 2,3,5
Example: If $\mathrm{a}: \mathrm{b}$ : c is $2: 5: 3$ and $\mathrm{c}: \mathrm{d}: \mathrm{e}$ is $2: 3: 5$ then find $\mathrm{a}: \mathrm{b}: \mathrm{c}: \mathrm{d}: \mathrm{e}$ ?
Sol.

| a | $:$ | b | $:$ | c | $:$ | d | $:$ | e |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | $:$ | 5 | $:$ | 3 | $:$ | 3 | $:$ | 3 |
| 2 | $:$ | 2 | $:$ | 2 | $:$ | 3 | $:$ | 5 |
| 4 | $:$ | 10 | $:$ | 6 | $:$ | 9 | $:$ | 15 |

Third proportional: If $2: 3:: 5: 7$, then 5 is called the third proportional to 2 and 3
Example: If $\mathrm{A}: \mathrm{B}$ is $2: 5, \mathrm{~B}: \mathrm{C}$ is $3: 4$, then find $\mathrm{A}: \mathrm{B}: \mathrm{C}$.
Sol. A : B $=2: 5$, $\mathrm{B}: \mathrm{C}=3: 4$ (given)

| A | $:$ | B | $:$ | C |
| :---: | :---: | :---: | :---: | :---: |
| 2 | $:$ | 5 | $:$ | 5 |
| 3 | $:$ | 3 | $:$ | 4 |
| 6 | $:$ | 15 | $:$ | 20 |

A: B:C=6:15:20

Mean proportional: Mean proportional between a and b is $\sqrt{a b}$
Ex. $A=2, b=3$ then mean proportional is $\sqrt{2 \times 3}=\sqrt{6}$
Comparison of Ratios: if we say that (a:b) $<$ (c:d) $=\frac{a}{b}<\frac{c}{d}$
Ex. $(2: 3)<(5: 7)=\frac{2}{3}<\frac{5}{7}$
Compounded ratio: The compound ratio of the ratios (a:b), (c:d), (e:f) is (ace:bdf)

Ex. (2:3), (5:7), (11:13)
$=(2 \times 5 \times 11):(3 \times 7 \times 13)=110: 273$

Duplicate ratio of ( $a: b$ ) is $\left(a^{2}: b^{2}\right)$
Ex. Duplicate ratio of $(2: 3)$ is $\left(2^{2}: 3^{2}\right)$
$=(4: 9)$
Example: If $\mathrm{P}: \mathrm{Q}=2: 3$ and $\mathrm{Q}: \mathrm{R}=4: 7$ then $(\mathrm{P}+\mathrm{Q})^{2}:(\mathrm{Q}+\mathrm{R})^{2}$ is
Sol.

| P | $:$ | Q | $:$ |
| :--- | :--- | :--- | :--- |
| 2 | R |  |  |
| 4 | 3 | $:$ | 3 |
| 4 | $:$ | 4 | $:$ |
| 8 | $:$ | 72 | $:$ |
| $(8+12)^{2}$ | $:(12+21)^{2}$ |  |  |
| $(20)^{2}:$ | $(33)^{2}$ |  |  |
| 400 | 1089 |  |  |

Sub-duplicate ratio of (a:b) is $(\sqrt{a}: \sqrt{b})$
Ex. Sub-duplicate ratio of $(2: 3)$ is $(\sqrt{2}: \sqrt{3})$
Triplicate Ratio of $(a: b)$ is $\left(a^{3}: b^{3}\right)$
Ex. Triplicate ratio of (2:3) is $\left(2^{3}: 3^{3}\right)$
$=(8: 27)$
Sub-triplicate ratio of (a:b) is $\left(a^{\frac{1}{3}:} b^{\frac{1}{3}}\right)$
Ex. $(2: 3)$ is $\left(2^{\frac{1}{3}}: 3^{\frac{1}{3}}\right)$
If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (componendo and dividendo)
Ex. $\frac{2}{3}=\frac{5}{7}$, then $\frac{2+3}{2-3}=\frac{5+7}{5-7}$
Variation: Two quantities A and B may be such that as one quantity changes in value, the other quantity also changes in value bearing certain relationship to the change in the value of the first quantity.

Example: P : Q : R along completed a piece of work in 20,40 and 30 days respectively. The ratio of the salary of each day is 5: 4: 3 respectively. If the total salary of $Q$ is Rs.144, find total salary of $P$.
Sol.

|  | P | $:$ | Q | $:$ | R |
| :---: | :---: | :---: | :---: | :---: | :---: |
| work | 20 | $:$ | 40 | $:$ | 30 |
| Salary | 5 | $:$ | 4 | $:$ | 3 |
|  | 100 | $:$ | 160 | $:$ | 90 |

=10: $16: 9$
$16 \mathrm{x}=144$
$\mathrm{x}=\frac{144}{16}$
$x=9$
$\mathrm{P}=10 \times 9=90$
Salary of $\mathrm{P}=$ Rs. 90


